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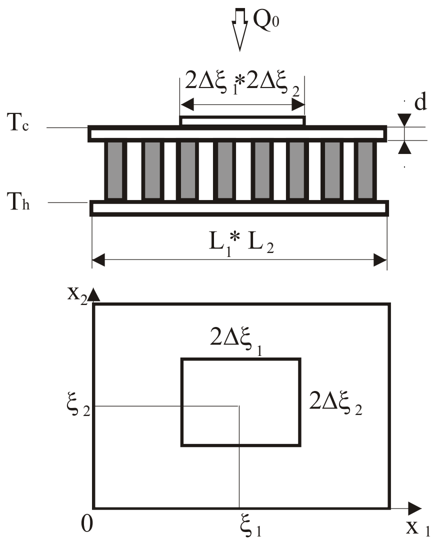
Temperature distribution on a thermoelectric cooler (TEC) cold surface is of high practical value, as the sizes of the cooled object may not coincide with the dimensions of the TEC cold side and it is necessary to make the object temperature closer to the average cold substrate temperature. It is also very important to take into account the temperature distribution on the intermediate substrates of multistage TECs both in mathematical simulation and design modeling.

The approach to finding the approximate two-dimensional temperature distribution for the case of a heat source located on the surface has been developed in papers [1,2]. In this paper this method is analytically verified and applied to calculations of the temperature 2D-profiles of the TEC substrates. Application of the above-mentioned method for performance improvement of TEC systems is discussed.

Consider a problem of the temperature distribution on the cold substrate surface of a single-stage TEC. Assume it consists of N pellets. There is a heat source localized on the TEC cold substrate.



Schematic view of the heat source on a single-stage TEC cold substrate surface

Suppose the TEC substrate is an L_1 x L_2 rectangular, and the heat source is a $2\Delta\xi_1$ x $2\Delta\xi_2$ one. The heat source centre coordinates are ξ_1, ξ_2. The heat source load to be pumped by the TEC equals Q_0. The hot surface temperature is a fixed value T_h, and the cold surface temperature is a two-dimensional funonalu the full substrate area per a pellet – L_1L_2/N. That is, we assume a quasi-continuous pellets distribution on the substrate surface. Within this approach the calculated temperature two-dimensional field differs from the real one by the absence of a slight periodicity (its period equals the distance between pellets). Then the pellets 2D-distribution density is equal to $\frac{N}{L_1L_2}$. Ignoring temperature dependences of thermoelectric parameters, we consider the Seebeck coefficient α, thermal conductivity κ and electrical resistivity ρ to be constant values. When the pellet is exposed to the electrical current I, the heat flux q_{pellet} [3] is pumped to the pellet cold end:

$$q_{pellet} = -\alpha I T_c + \frac{1}{2} I^2 R + k(T_h - T_c), \tag{1}$$

where the first term on the right side of equation (1) expresses the Peltier heat extracted by the pellet from the substrate, the second term is the part of the Joule heat, arriving at the substrate from the pellet, and the third term describes the heat flux coming from the hot substrate by the pellet thermal conductance. Here α – the Seebeck coefficient, $R = \rho \frac{l}{s_0}$ – pellet electrical resistance, $k = \kappa \frac{s_0}{l}$ – pellet thermal conductance, l – pellet length. Let d denote the substrate thickness and λ stand for the substrate thermal conductivity. Then the heat conductance equation can be written as follows:

$$\lambda d\left(\frac{\partial^2 T_c}{\partial x_1^2}\right) + \lambda d\left(\frac{\partial^2 T_c}{\partial x_2^2}\right) - \frac{N(\alpha I + k)T_c}{L_1L_2} + \frac{N\left(\frac{1}{2} I^2 R + kT_h\right)}{L_1L_2} + \frac{Q_0 1\{u\}}{4\Delta\xi_1\Delta\xi_2} = 0, \tag{2}$$

where we write the symbol 1{u} for the function equal 1 within the area of the heat source Q_0 and 0 within the rest of the surface. Suppose the heat is only removed from the cold substrate by the pellets and there are no lateral heat fluxes:

$$\left.\frac{\partial T_c}{\partial x_i}\right|_{x_i=0,\, x_i=L_i} = 0, \quad i = 1, 2. \tag{3}$$

If turning the current I into the reduced current $j = \frac{Il}{s_0}$ and denoting the pellets filling coefficient K_f:

$$K_f = \frac{Ns_0}{L_1L_2}, \tag{4}$$

we define:

$$b^2 = \frac{(\alpha j + \kappa)K_f}{\lambda l d}, \tag{5}$$

$$A = \frac{K_f\left(\frac{1}{2} j^2 \rho + \kappa T_h\right)}{\lambda l d}, \tag{6}$$

$$C = \frac{Q_0}{4\Delta\xi_1\Delta\xi_2 \lambda d}. \tag{7}$$

Making in (2) the substitution of variables

$$T_c = \theta + \frac{A}{b^2}, \qquad (8)$$

we obtain the following equation:

$$\frac{\partial^2\theta}{\partial x_1^2} + \frac{\partial^2\theta}{\partial x_2^2} - b_1^2\theta + C_1 1\{u\} = 0 \qquad (9)$$

with boundary conditions:

$$\left.\frac{\partial\theta}{\partial \bar{x}_i}\right|_{\bar{x}_i=0,\bar{x}_i=1} = 0, \qquad i = 1,2. \qquad (10)$$

The approximate solution of this problem is known and given in papers [1,2]:

$$\theta = \frac{C_1}{b_1^2}\phi_1\phi_2. \qquad (11)$$

In dimensionless coordinates

$$\bar{x}_i = \frac{x_i}{L_i}, \bar{\xi}_i = \frac{\xi_i}{L_i}, \Delta\bar{\xi}_i = \frac{\Delta\xi_i}{L_i}, \qquad i = 1,2 \qquad (12)$$

the functions ϕ_i have the view:

$$\phi_i = \begin{cases} K_i ch(p_i\bar{x}_i), \ x_i \in [0; \bar{\xi}_i - \Delta\bar{\xi}_i] & K_i = \dfrac{2sh(p_i\Delta\bar{\xi}_i)ch(p_i(1-\bar{\xi}_i))}{shp_i} \\ K_i ch(p_i\bar{x}_i) - ch(p_i(\bar{x}_i - \bar{\xi}_i + \Delta\bar{\xi}_i)) + 1, \ x_i \in [\bar{\xi}_i - \Delta\bar{\xi}_i, \bar{\xi}_i + \Delta\bar{\xi}_i] & \\ K_i ch(p_i\bar{x}_i) - ch(p_i(\bar{x}_i - \bar{\xi}_i + \Delta\bar{\xi}_i)) + ch(p_i(\bar{x}_i - \bar{\xi}_i - \Delta\bar{\xi}_i)), \ x_i \in [\bar{\xi}_i + \Delta\bar{\xi}_i, 1] & \end{cases}$$

i = 1,2, (13)

where

$$p_i = \frac{L_i}{L_{i+(-1)^{i-1}}}\sqrt{B_{i+(-1)^{i-1}}\left[1{,}5 - \left(\frac{sh\left(2\left(\sqrt{B_{i+(-1)^{i-1}}}\right)\right)}{2\left(\sqrt{B_{i+(-1)^{i-1}}}\right)} + 1\right)^{-1}\right]}, \ i = 1, 2, \qquad (14)$$

$$B_i = b^2 L_i^2, \qquad i = 1,2. \qquad (15)$$

(e)34 Therefore, the temperature distribution for the case in F

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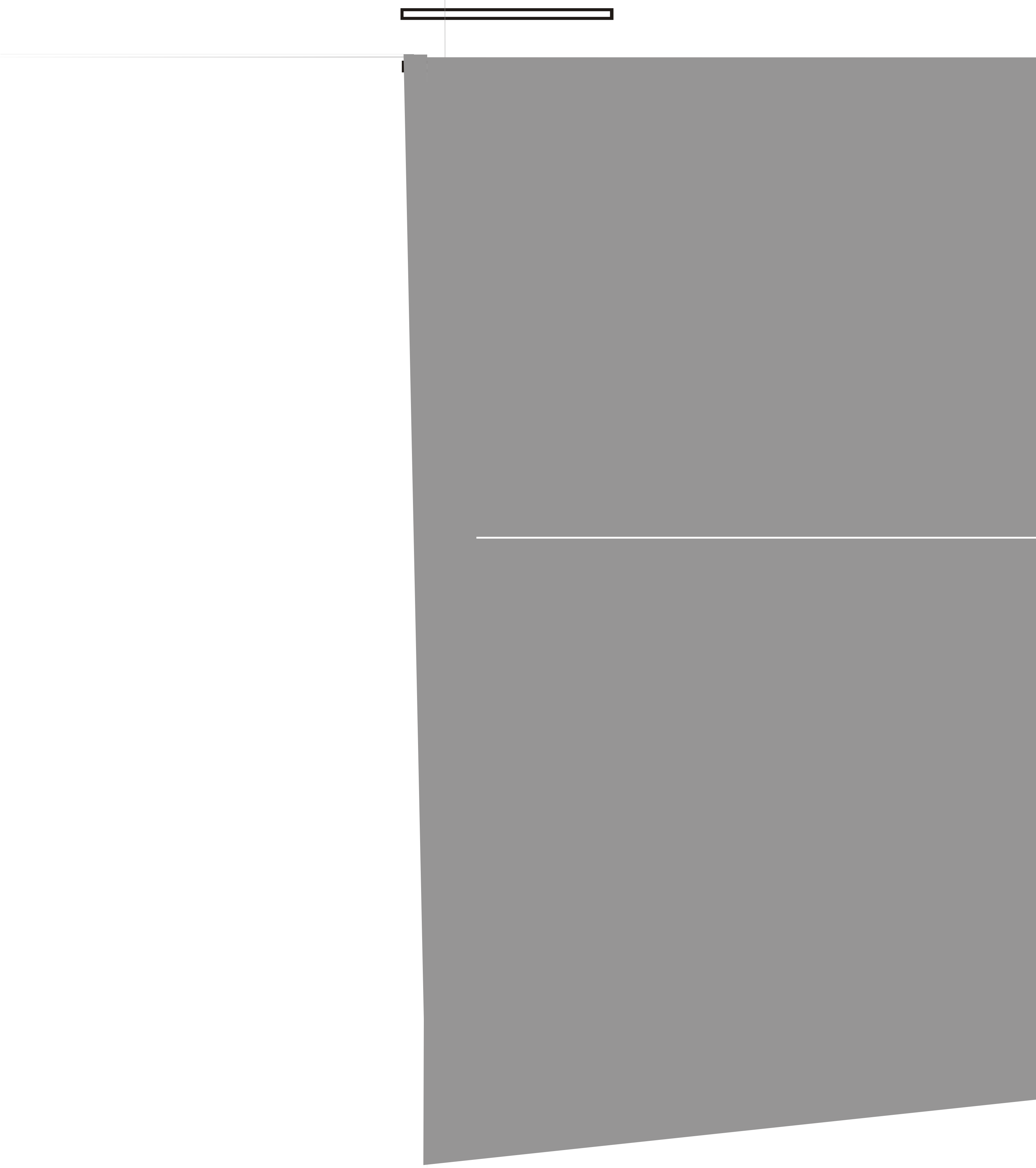
its stage index. The current densities for two stages can also differ and we denote them j_i, i = 1,2. Our objective is obtaining temperature distribution $T_c(x,y)$ on the intermediate substrate between the first and the second cascade.



$$c=\frac{K_{f1}}{\lambda dl_1}\left[\frac{\alpha_1^2 j_1^2\overline{T}_c+\frac{1}{2}j_1^2\rho_1(\alpha_1 j_1+2\kappa_1)+\kappa_1 q_0\frac{l_1}{N_1 s_1}}{\alpha_1 j_1+\kappa_1}\right], \tag{21}$$

$$a=\frac{K_{f2}\left(\frac{1}{2}j_2^2\rho_2+\kappa_2 T_h\right)}{\lambda dl_2}, \tag{22}$$

$$\beta^2=\frac{K_{f2}(\alpha_2 j_2+\kappa_2)}{\lambda dl_2}. \tag{23}$$

As a result of these transformations the value $\overline{T}_c$ remains unknown. It can be found by a multi-iteration procedure. For a zero approximation we take $\overline{T}_c$ as a solution of linear equations, describing heat balance on the TEC substrate without allowing for heat losses and temperature distribution (that is, in the one-dimensional approach):

$$\overline{T}_c=\frac{4\Delta\xi_1\Delta\xi_2}{L_1 L_2}\frac{c}{\beta^2}+\frac{a}{\beta^2}. \tag{24}$$

The solution is

$$\overline{T}_c=\frac{\frac{N_2 s_2}{l_2}\left(\frac{1}{2}j_2^2\rho_2+\kappa_2 T_h\right)(\alpha_1 j_1+\kappa_1)-\frac{N_1 s_1}{l_1}\frac{1}{2}\alpha_1 j_1^3\rho_1+q_0\kappa_1}{\frac{N_2 s_2}{l_2}(\alpha_2 j_2+\kappa_2)(\alpha_1 j_1+\kappa_1)-\frac{N_1 s_1}{l_1}\alpha_1^2 J_1^2} \tag{25}$$

After the temperature distribution is found in the first iteration, we can carry out the integration over the thermal contact area and calculate $\overline{T}_c$. As the solution of the heat conduction equation is expressed in the analytical form, the correspondent integrals are easily calculated. If denoting

$$\varphi_i=\frac{\int_{\bar{\xi}_i-\Delta\bar{\xi}_i}^{\bar{\xi}_i+\Delta\bar{\xi}_i}\phi_i(x)dx}{2\Delta\bar{\xi}_i}=\frac{\left[2K_i\left(chp_i\bar{\xi}_i shp_i\Delta\bar{\xi}_i\right)-sh2p_i\Delta\bar{\xi}_i\right]}{2p_i\Delta\bar{\xi}_i}+1,\ i=1,2, \tag{26}$$

the expression for $\overline{T}_c$ can be written as follows:

$$\overline{T}_c=\frac{c}{\beta^2}\varphi_1\varphi_2+\frac{a}{\beta^2}. \tag{27}$$

With the help of Eq. (27) we can find the value $\overline{T}_c$, calculate a new value of c_1 from Eq. (21) and, with it, find a new $\overline{T}_c$ and etc. The procedure described above converges quickly and only a few iterations are required.

Due to the heat losses the average temperature of the thermal contact area $\overline{T}_c$ is different from the average temperature of the whole intermediate substrate $\overline{T}_{cl}$:

$$\overline{T}_{cl}=\frac{c_1}{\beta_1^2}\zeta_1\zeta_2+\frac{a_1}{\beta_1^2}, \tag{28}$$

where we write ζ_i for the following:

$$\zeta_i=\int_0^1\phi_i(x)dx=2\Delta\bar{\xi}_i,\quad i=1,2. \tag{29}$$

Eq. (29), taken into account Eq. (28), coincides with Eq. (24), i. e. the temperature (24), used in the first iteration, is exactly the average over the whole substrate area. If the difference between $\overline{T}_c$ and $\overline{T}_{c1}$ is slight, one iteration may be enough.

The above formulae allow calculating the temperature distribution over the substrates of a single-stage and multistage TEC. In fact, to perform this calculation it is sufficient to be capable of finding the temperature distribution on the cold substrate of a single-stage TEC (see (16)), as evaluating the operational heat load on a TEC stage is a standard task of a TEC mathematical simulation.

Eq. (28) may be applied not only for a two-stage TEC, but also for a second stage of a multicascade TEC. For this purpose one has to know, even approximately, the temperature of the second stage hot substrate. Once the heat rejected by the previous cascades is found, it is possible to calculate the temperature distribution on any stage cold substrate of a multicascade TEC with the help of Eq. (28).

In practice it is often more important not to obtain the temperature two-dimensional field of the substrate but its average temperature $\overline{T}$ and the average temperature $\overline{T}_q$ of the contact area under the heat load. As an appropriate criterion of the distributional uniformity we take the difference $\Delta\overline{T} = \overline{T}_q - \overline{T}$. The analytical form of the heat conductance equation allows finding $\Delta\overline{T}$ easily. Thus we come to the following equation for a single-stage TEC:

$$\Delta\overline{T}_1 = \frac{Q_0 l}{\overline{S}_q K_f (L_1 L_2)(\alpha j + \kappa)} \left(\varphi_1\varphi_2 - 4\Delta\overline{\xi}_1\Delta\overline{\xi}_2\right) \qquad (30)$$

and for a two-stage TEC:

$$\Delta\overline{T}_2 = \frac{c_1}{\beta_1^2}\left(\varphi_1\varphi_2 - 4\Delta\overline{\xi}_1\Delta\overline{\xi}_2\right). \qquad (31)$$

As examples we give the results of $\Delta\overline{T}$ calculation for various kinds of heat sources localized on the cold side of the standard 127-couple 40x40 mm^2 TEC (1.15x1.4x1.4 mm^3 pellets) for different materials of the cold substrate (Table 1). The hot substrate temperature is taken 300 K. The reduced electric current is j=20 A/см. The heat source is placed in the centre of the cold substrate (ξ_1=20mm, ξ_2=20mm).

The difference $\Delta\overline{T}$ of the temperature averaged over the area of the heat source and the temperature averaged over the whole substrate of the 127-couple 40x40 mm^2 TEC.

		Δξ x Δξ				$\Delta\overline{T}$ К
1	10	10x10	1	Al_2O_3	30	40.3
2	10	10x10	1	AlN	150	8.5
3	10	20x20	1	Al_2O_3	30	14.2
4	10	20x20	1	AlN	150	3.5
5	10	30x30	1	Al_2O_3	30	3.3
6	10	10x10	2	Cu	400	1.3

The data in Table 1 shows that the localized heat load with the heat density 10 W/cm^2 is poorly spread over the substrate of Al_2O_3. In this case even the AlN ceramics is not a way out. Only the 2mm thick copper substrate allows reducing heat losses to the extent of the calculation errors.

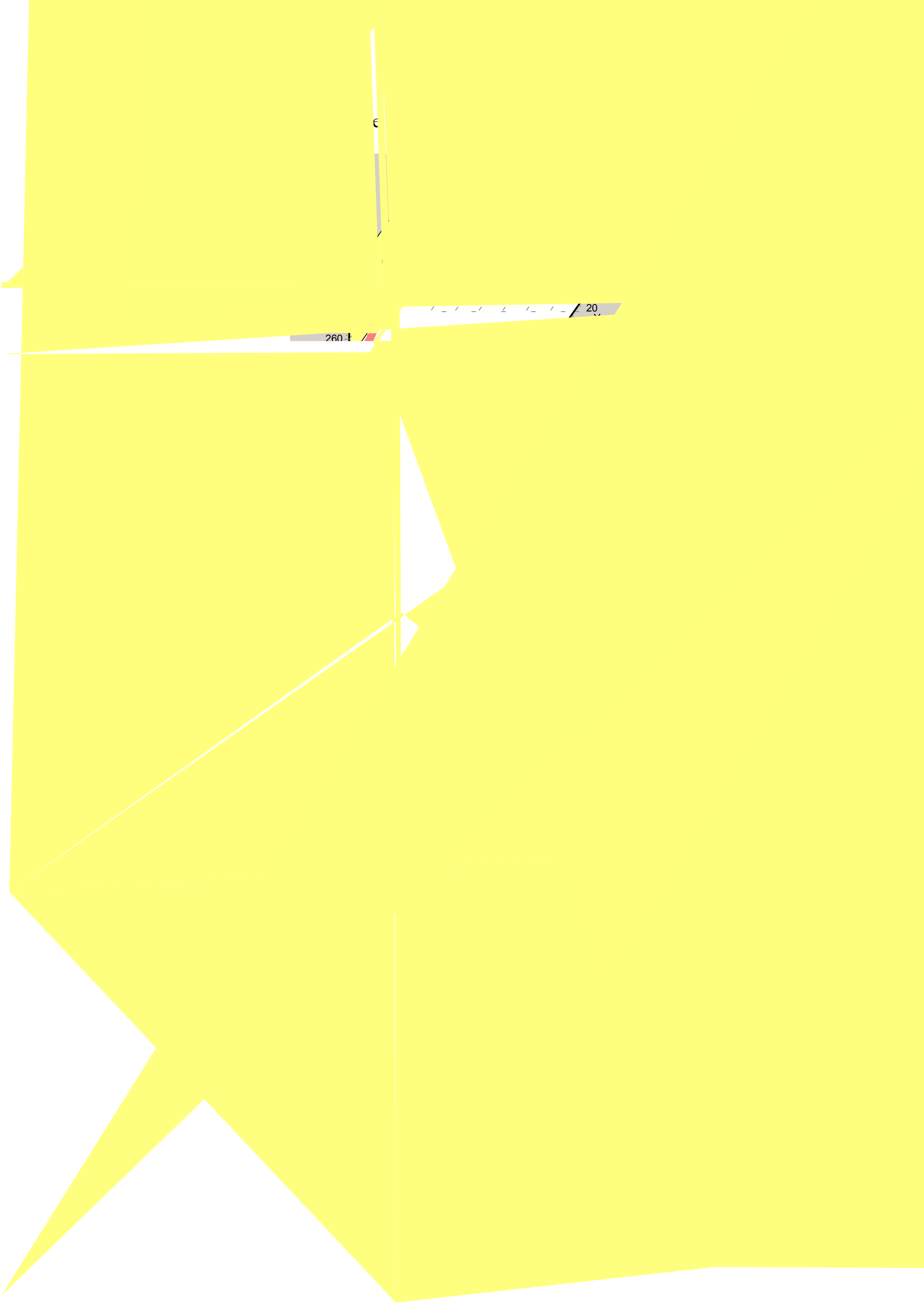
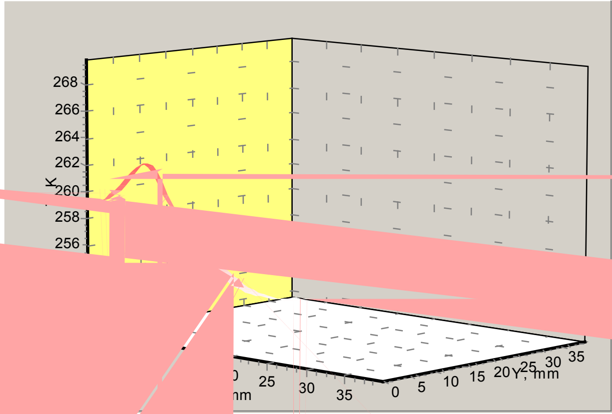

260
20

Figure 4 Example of 2D-temperature field for non-central localization of the non-square heat source: heat source 10 W, are



In TEC design manufacturing a theoretical modeling should be of great reliability. Nowadays it is no problem

substrate of two-cascade TEC's 1 and 2

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